

I hope the theory of arithmetic mean we discussed last week is clear to you. Let's see the theory in action today. I will pick some mean questions from various sources (Official Guide, GMAT prep tests, etc.) and we will try to use the concepts we learned last week to solve them.

Let's start with a simple question.

Question 1: For the past n days the average daily production at a company was 60 units. If today's production of 100 units raises the average to 65 units per day, what is the value of n ?

- (A) 30
- (B) 18
- (C) 10
- (D) 9
- (E) 7

Solution: If today's production were also 60 units, what would have happened to the average? Obviously, it would have stayed the same! But today's production is 40 units extra and hence it raised the average. It raised the average by 5 units which means that each one of the n observations and today's observation got an extra 5. Since 40 got distributed and each was given 5, there must have been a total of $40/5 = 8$ observations including today's. Therefore, the value of n must have been $8 - 1 = 7$.

Answer (E)

I know you can solve the question using the formula of averages. In fact, you can solve every question using the formula and working out the values. But the point is that the logical method helps you solve the question very quickly and you are less likely to make calculation errors since there aren't too many calculations to perform! Let's go on now.

Question 2: When Anna makes a contribution to a charity fund at school, the average contribution size increases by 50%, reaching \$75 per person. If there were 5 other contributions made before Anna's, what is the size of her donation?

- (A) \$100
- (B) \$150
- (C) \$200
- (D) \$250
- (E) \$450

Solution: After Anna's contribution, the average size increases by 50% and reaches \$75. What must have been the average size of contribution before Anna's donation? It must have been \$50 since a 50% increase would lead us to \$75. So, \$50 was the average size of 5 donations before Anna made her donation. Had Anna donated \$50 as well, the average would have stayed the same i.e. \$50. But the average increased to \$75 which means that Anna donated an extra \$25 for each of the 6 observations (including her) in addition to the \$50 she would have donated to keep the average same.

Hence, the amount Anna donated = $50 + 6 \times 25 = \$200$

Answer (C)

Again, this was a relatively straight forward question. Let's look at a tricky one now.

Question 3: A set of numbers has an average of 50. If the largest element is 4 greater than 3 times the smallest element, which of the following values cannot be in the set?

- (A) 85
- (B) 90
- (C) 123
- (D) 150
- (E) 155

Solution: This question might look a little ominous but it isn't very tough, really! The set has an average of 50 so that already tells us that we can represent each element of the set by 50. If there is an element which is a little less than 50, there will be another element which is a little more than 50.

The largest element is 4 greater than 3 times the smallest element so $L = 4 + 3S$.

The smallest element must be less than 50 and the largest must be greater than 50. Say, if the smallest element is 20, the largest will be $4 + 3 \cdot 20 = 64$.

Is there any limit imposed on the largest value of the largest element? Yes, because there is a limit on the largest value of the smallest element. The smallest element must be less than 50. The smallest member of the set can be 49.9999... The limiting value of the smallest number is 50. As long as the smallest number is a tiny bit less than 50, you can have the greatest number a tiny bit less than $4 + 3 \cdot 50 = 154$. The number 154 and all numbers greater than 154 cannot be a part of the set. Say if the smallest element is 49, the largest element will be $4 + 3 \cdot 49 = 151$. So the set could look something like this:

$S = \{49, 49, 49, 49, \dots (101 \text{ times to balance out the extra } 101 \text{ in } 151), 50, 50, 151\}$

Only option (E) cannot be a part of the set.

These were some of the basic (and not so basic) questions of mean that we could come across in GMAT. Let's call it a day now. We will look at some more stats concepts next week. Till then, keep practicing!